Radiation Resistance of Inverted V Antennas

And Some Observations on Resonant Length

BY DALE W. COVINGTON,* K4GSX

This theoretical and experimental study of the feed characteristics of the inverted V antenna supplements the same author's earlier analysis (*QST*, May, 1965) in which the radiation patterns of the antenna were worked out. Together, the two articles give a comprehensive picture of the effect of bending a dipole at its center.

"ES ANT IS INVERTED V ABT — FT ABOVE GND . . ." Phrases such as this form an integral part of many 80- and 40-meter QSOs. The following discussion deals specifically with the radiation resistance of this popular antenna as a function of the antenna's height above ground. With such information it becomes an easy matter to select combinations of V heights and angles which approximately match the antenna resistance with the proper transmission-line characteristic impedance, or to select a transformer to go between the antenna and the feed line.

Technique

A basic problem in calculating the input impedance of the inverted V rests in the fact that the antenna is commonly used on the lower frequency bands, and hence is usually installed just fractions of a wavelength above ground possessing finite conductivity and a dielectric constant other than 1. Thus the V analysis described by King[1] for the isolated antenna is of limited value. Even if the case of perfectly-conducting ground is assumed, a strictly formal treatment of the problem remains quite difficult as the calculations must account for that part of the total current in the antenna which is induced by current flow on the ground plane.

On the other hand, a rather straightforward numerical solution can be obtained by utilizing a specialization of the method of moments.[2] Here the radiated power including the mutual radiation terms along with the self-impedance terms is evaluated from a spatial distribution of small current elements representing the antenna and its image. The radiation resistance follows from the computed value of the radiated power, once a current distribution is assigned to the antenna. Such was the approach taken in the present analysis.

* 281 Vance Circle N.E., Marietta, Georgia 30060

[1] King, *Theory of Linear Antennas*, University Press, Cambridge, Massachusetts, 1956, p. 389.

[2] Schelkunoff and Friis, *Antennas, Theory and Practice*, John Wiley and Sons, Inc., New York, 1952, p. 162.

Of course, the antenna *impedance* includes an inductive term which varies with V height, angle, and conductor length/diameter ratio. For the isolated V this component ranges between 15 and 40 ohms for included V angles between roughly 90 and 180 degrees [1]. By shortening the antenna slightly the inductive reactance is reduced to zero, leaving only the more slowly-varying real component. Consequently the antenna resistance measured at resonance is slightly smaller than the real part of the input antenna impedance or actual radiation resistance.[3]

Fig. 1 illustrates the general model and nomenclature. The inverted V symmetrically located over perfectly conducting ground is replaced by current elements positioned in the center of ⅛-wavelength intervals along the antenna and its image. The elements are weighted by the assumed cosinusoidal current flowing on the thin wire filaments. Certainly, subdividing the antenna into more than four current elements would add both toil and accuracy to the computation. As will be shown in the next section, however, the even cruder two-element approximation, with just one current element in the center of each leg of the antenna, compares well with the radiation resistance as determined by a continuous current distribution.

Computed Results

The initial calculations considered the self- and mutual-resistance terms for the current elements of the antenna alone. This effectively represented the isolated V far removed from ground, and afforded an immediate check on the

[3] *The ARRL Antenna Book.*

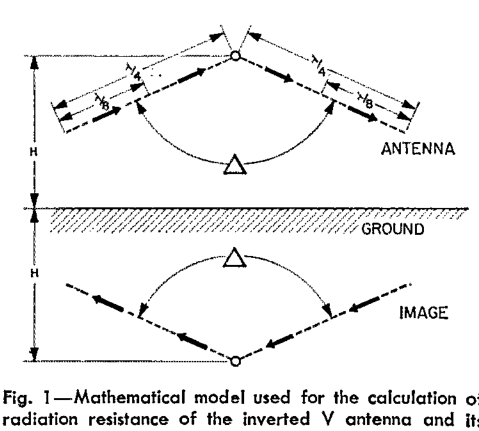

Fig. 1—Mathematical model used for the calculation of radiation resistance of the inverted V antenna and its image in perfectly conducting ground.

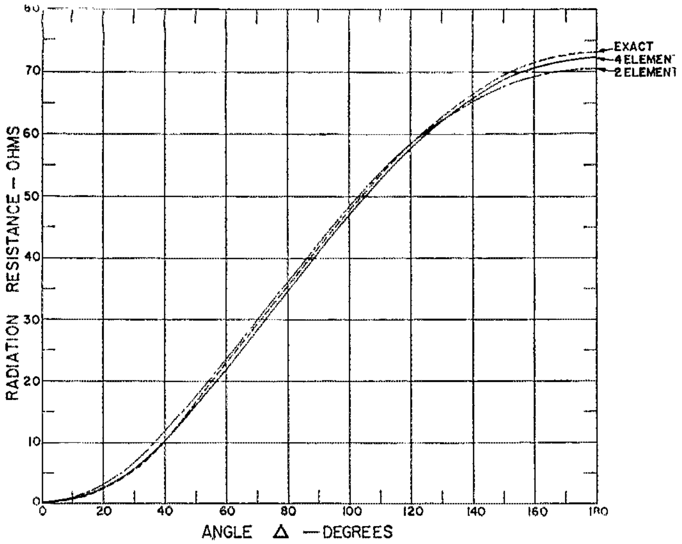

Fig. 2—Calculated free-space radiation resistance as a function of delta. Solid curve—model using four current elements as in Fig. 1; Dot-dash curve—model using two current elements; dotted curve—continuous current distribution.

accuracy of the model. A plot of the radiation resistance versus V angle is given in Fig. 2. The solid line and the broken line correspond to the four- and two-current-element models respectively. The dotted line is taken from King's results for a continuous current distribution. Agreement between the three cases is good. Note the rapid and almost uniform 0.6 ohm per degree decrease in radiation resistance as the V angle sharpens from 130 to 30 degrees. The radiation resistances at the extreme values of delta are intuitively correct; namely, 73 ohms for delta of 180 degrees and zero when the legs of the antenna are parallel.

The image antenna was introduced in the second set of calculations. Fig. 3 portrays the dependence of the radiation resistance upon the apex height above perfect ground. Again a reference benchmark was available for delta equal to 180 degrees. The dotted line in Fig. 3 indicates the influence of ground on the horizontal dipole radiation resistance.[3,4] As delta decreases, the radiation resistance initially begins decreasing rather slowly, but for angles smaller than 120 degrees the radiation resistance quickly falls to fairly low values. In fact, for delta much smaller than 90 degrees some form of step-up transformer would be required for best matching to 52-ohm coax. Interestingly enough, 73-ohm coax suitably coupled to the balanced antenna appears to be a good feed-line choice for a wide range of delta at heights near one-half wavelength. Increasing the apex height finds the radiation resistance for a particular delta indeed oscillating about the corresponding isolated antenna values given in Fig. 2.

Like most theoretical curves based on simple models, the curves of Fig .3 must be evaluated with a grain of salt when compared to the real antenna world. It will be useful to draw upon past experience with horizontal dipoles to define the character of the approximations involved in the results of Fig. 3. First, an element of approximation is introduced by equating the antenna resistance measured at resonance to the radiation resistance, particularly as the antenna deviates from the thin-wire type. When the perfect ground of the model is replaced by actual ground with finite conductivity, the magnitude and phase of the image-current elements must be suitably altered in accord with the

[4] Kraus, *Antennas*, McGraw Hill, New York, 1950, p. 305

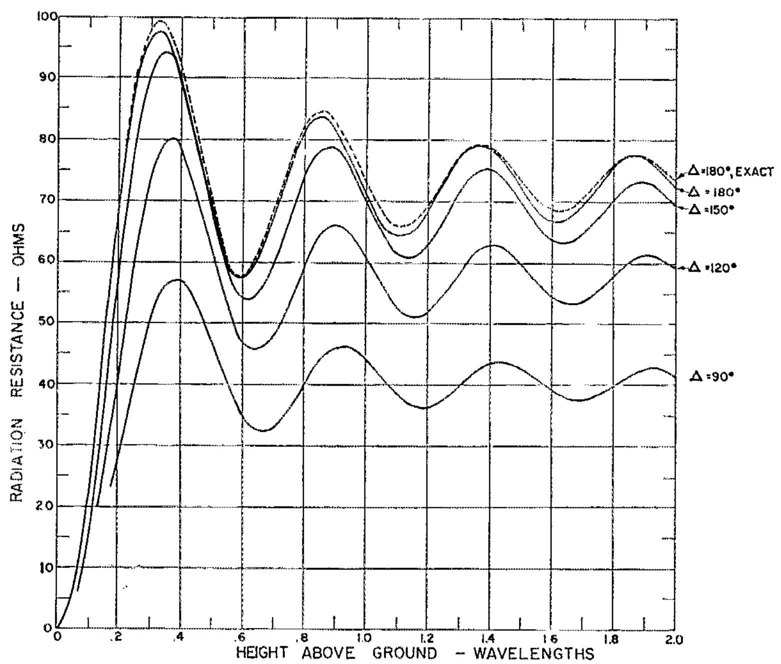

Fig. 3—Radiation resistance vs. height above perfectly-conducting ground for representative values of delta. Dotted curve is the variation in radiation resistance of a horizontal half-wave dipole as usually given in texts.

ground reflection coefficients. Real ground has the effect for horizontal dipoles of shifting the entire curve slightly to the left and reducing the amplitude of its oscillations.[5] Finite ground conductivity also causes the radiation resistance to increase instead of dropping to zero as the dipole height falls below 0.2 wavelength.[6] Perhaps this is the explanation for Johnson's figure of 82 ohms for a 145-degree V at a height of 0.114 wavelength.[7] No increase was observed experimentally for the sharper V's described in this note.

Measured Data

It seemed appropriate to press beyond the computed curves of the model into some actual experimental results. Measurement of the radiation resistance of an 80-40 meter cage V[8] using a homemade bridge[9] yielded the values shown below.

Ant.	Angle Δ	Height	R_{RAD}
80 m.	110°	.17λ	38 ohms
40 m.	95°	.32λ	54 ohms

To check the resistance variation over ground, an inverted V with delta of 105 degrees was constructed for 20 meters using ⅞-inch aluminum tubing. This gave an L/D ratio that was similar to the cage antenna. A G3HZP balun[10] was inserted between the RG-58 feed line and the antenna terminals. Fig. 4 contains both the experimental 20-meter data and the curves for a 105-degree V computed from the 2- and 4-current-element models. Certainly at the higher elevations these curves should bracket the radiation resistance as determined by a continuous current on the antenna. The points denoted by triangles were measured on the homemade bridge and the circled points were obtained using a u.h.f. GR Type 1602 bridge. While the experimental points do not fall on the computed curves, they do cluster in a range of values consistent with the previous observations on antennas in theory and in practice.

Length of the Inverted V

The pruning necessary to tune out the reactance of the half-wave antenna can be estimated by introducing a variable K into the half-wavelength formula.

$$Length\ (feet) = \frac{492 \times K}{f\ (Mc.)}$$

Into K has been lumped information on the influence of the conductor diameter, loading by end insulators, height above ground of arbitrary conductivity, feed-line effects, etc. Normally, a representative value of 0.95 is assigned to K, which immediately leads to the familiar formula

[5] Jordan, *Electromagnetic Waves and Radiating Systems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1950, p. 524.

[6] Proctor, "Input Impedance of Horizontal Dipole Aerials at Low Heights Above the Ground", *Proc. I.E.E.*, *97*, part II, 1950, p. 188.

[7] Johnson, "Antenna Feedpoint Investigation", *CQ*, March 1968, p. 36.

[8] Covington, "Inverted V Radiation Patterns", *QST*, May 1965, p. 81.

[9] Strandlund, "Amateur Measurement of R + jX", *QST*, June 1965, p. 25.

[10] James, "The G3HZP Balun", *RSGB Bulletin*, July 1966, p. 459.

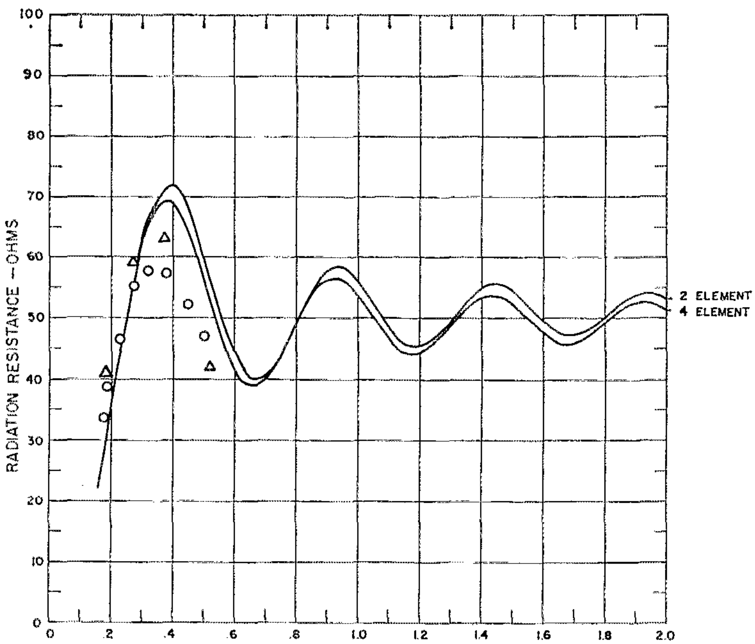

Fig. 4—The triangles and circles represent measured values of radiation resistance of an experimental inverted V having an included angle, delta, of 105 degrees. Curves based on two mathematical models are shown for comparison.

for the length of the horizontal resonant half wavelength antenna.

$$Length\ (feet) = \frac{468}{f\ (Mc.)}$$

The very geometry of the inverted V indicates that some limitations must be accepted in assuming a single, universal number for K. Moreover this is confirmed by the maze of conflicting values for resonant lengths of inverted Vs reported in the literature. Some of these reports can be untangled by focusing attention on K in relation to the antenna height.

A 10-meter balun-fed inverted V was built with delta of 105 degrees and an L/D ratio of 230. The resonant frequency was determined from the minimum of the s.w.r. plots taken as the antenna was raised to various heights above ground. This data was then used to compute K as given by the first formula. The results are shown in Fig. 5. Perhaps the most surprising aspect of Fig. 5 was the discovery that, given the antenna height, the value of K read from the curve predicted (using the first formula) the resonant lengths of both the 80- and 40-meter cage Vs to within 1.6 feet. Several tentative conclusions are suggested by the figure. The initial rise followed by an oscillatory behavior is similar to that noted for horizontal dipoles.[6] Larger angles of delta should witness a steeper decline in K at the lower heights. As the distance above ground increases, K assumes a value that is slightly larger than the corresponding value for a dipole. This lends weight to the argument that the dipole has the larger reactive component. Using a d.c. three-ground-rod technique, the local ground conductivity was found to be 2.7 millimhos/meter under the antenna. Since the conductivity over much of the U.S. is several times greater than this [11] it would be reasonable to expect, in general, values of K which move into the 0.95 region more quickly than Fig. 5 indicates. A larger L/D ratio should increase K slightly.

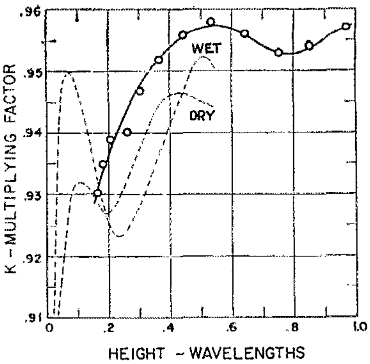

Fig. 5—Experimentally determined values of K at various heights above ground. The circled points were measured for a 10-meter, 105-degree inverted V with L/D of 230 and ground conductivity of 2.7 millimhos/meter. The dotted lines were taken from Proctor's 6-meter, horizontal-dipole results (L/D of 390) over wet and dry ground.[6]

Summary

Basically, this study has examined two questions:

1) What is the most efficient way of feeding an inverted V given an arbitrary V height and angle? Fig. 3 supplies an approximate answer in the case of a thin-filament V and perfect ground. Practically speaking, the framework of the idealized curves was confirmed experimentally. A more exact confirmation would require a better mathematical model of the physical antenna and ground. Nevertheless Fig. 3 does suggest some guidelines for feeding inverted Vs. The simplest approach, particularly for multiband operation, is to use tuned feeders and not really worry about the antenna impedance. But tuned feeders do not offer the convenience and portability of coaxial or Twin-Lead feed. In this case the best match will depend on height and V angle. With 73-ohm coax or 75-ohm Twin Lead an s.w.r. of 1.7 or better at resonance is predicted for any height above 0.25 wavelength and delta greater than 120 degrees. Roughly the same could be said for a 300-ohm folded-dipole inverted V. Feeding with 52 ohm-coax requires judicious selection of height and delta if the lowest s.w.r. is to be maintained. The standard deviation of the s.w.r. about 52 ohms for heights from 0.2 to 1 wavelength is minimum for delta near 110 degrees. This would represent a good compromise for a multiband trap inverted V. The purist approach to the feed problem would be to select the antenna height that optimizes the radiated power for a given set of angles of elevation, then to match the antenna impedance at that height to the coax using a balun transformer.

2) Given a frequency, how long is a corresponding inverted V antenna? With much sagacity comes the three-word reply, "cut and try." While an inverted V installed high and in the clear can be longer than a horizontal dipole, especially if delta is less than 90 degrees, the cramped conditions prevailing for the usual 80/40-meter antenna will cause lengths computed by the $468/f$ formula to be too long. The author has found Fig. 5 to be a useful supplemental guide in zeroing in on the resonant frequency for Vs near 100 degrees.

Correspondence with G3HZP has been especially valuable and much of the prerequisite enthusiasm and elbow grease in all phases of the experimental measurements came from W4LKB. Special thanks go to both amateurs for their help in the presentation of the above ideas on inverted Vs. QST

[11] *FCC Rules and Regulations*, Section 73, p. 107.